Sub. Code: 007

NEB - GRADE XI

Mathematics

Model Question - 2077

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks

Time - 3 hrs. **Full Marks – 75**

Attempt **all** the questions.

Group 'A' **(1 × 11 = 11)**

Rewrite the correct option in your answer sheet

1. Which of the following is a statement?

(a) The fishes are beautiful. **(b) Study mathematics.**

(c) x is a capital of country y. **(d) Water is essential for health.**

✍ (d) Water is essential for health.

2. The value of: $\sqrt{-16}\times\sqrt{-25}$ is

(a) – 20 **(b) – 20i** **(c) 20i** **(d) 20**

✍ (a) – 20

Explanation: $\sqrt{-16}\times\sqrt{-25}=\sqrt{16i^2}\times\sqrt{25i^2}=4i\times 5i=20i^2=-20$

(**Note:** In exam, no need to give explanation for MCQs.)

3. If $\angle C=60^o$, b = 5 cm and a = 4 cm of ΔABC, what is the value of c?

(a) 3.58 cm **(b) 4.58 cm** **(c) 4.89 cm** **(d) 4.56**

✍ (b) 4.58

Explanation: $c=\sqrt{a^2+b^2-2ab\cos C}=\sqrt{4^2+5^2-2\times 4\times 5\times\cos 60^o}=4.58$

4. In a triangle ABC, $\angle B=120^o$, a = 1 and c = 1 then the other angles and sides are

(a) 35, 45, $\sqrt{2}$ **(b) 10, 50, $\sqrt{3}$** **(c) 20, 40, 2** **(d) 30, 30, $\sqrt{3}$**

✍ (d) 30, 30, $\sqrt{3}$

Explanation: Here, $a=c=1$, so ΔABC is an isosceles triangle. In an isosceles triangle base angles are equal. The only option in which base angles are equal is option (d)

5. The cosine of the angle between the vectors $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$ and $\vec{b}=\vec{i}+3\vec{j}+3\vec{k}$ is

(a) $\frac{1}{14}$ **(b) 14** **(c) $\sqrt{14}$** **(d) 196**

✍ None of the option. Option given is mistake.

Explanation: $\cos\theta=\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\frac{1\times1-2\times3+3\times3}{\sqrt{1^2+(-2)^2+3^2}\times\sqrt{1^2+3^2+3^2}}=\frac{4}{\sqrt{14}\times\sqrt{19}}=0.25$

6. The equation of parabola with the vertex at the origin and directrix $y-2=0$ is
(a) $x^2-8y=0$ **(b) $y^2+8y=0$** **(c) $x^2+8y=0$** **(d) $y^2-8y=0$**

✍ (c) $x^2+8y=0$

Explanation: For vertex at origin and directrix $y=a$, the equation of parabola is $x^2=-4ay$

7. A mathematical problem is given to three students Sumit, Sujan and Rakesh whose chance of solving it are $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{a}$ respectively. The probability that the problem is solved is $\frac{3}{4}$. The possible values of a are

(a) $\frac{9}{2}$ **(b) 4** **(c) $\frac{1}{4}$** **(d) $\frac{1}{8}$**

✍ (b) 4

Explanation: $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{a}\right)=1-\frac{3}{4}$

$$\Rightarrow\frac{1}{2}.\frac{2}{3}.\left(1-\frac{1}{a}\right)=\frac{1}{4}$$
$$\Rightarrow1-\frac{1}{a}=\frac{3}{4}$$
$$\Rightarrow\frac{1}{a}=\frac{1}{4}$$
$$\Rightarrow a=4$$

8. $\lim\limits_{\theta\to0}\frac{\sin\theta}{\theta}$ is equal to

(a) 0 **(b) ∞** **(c) 1** **(d) $\frac{0}{0}$**

✍ (c) 1

9. The derivatives of $\frac{4x^2+3}{3x^2-2}$

(a) $\frac{-34x}{(3x^2-2)^2}$ (b) $\frac{30x^2}{3x^2-2}$ (c) $\frac{-32x}{(3x^2-2)^3}$ (d) $\frac{-31x}{(3x-2)^2}$

✍ (a) $\frac{-34x}{(3x^2-2)^2}$

Explanation: We know, $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$. So, at first, we search v^2 in denominator in the given options. The only option that satisfies this condition is option (a). In this way, you can solve given question faster and easier than solving as below.

$$\frac{d}{dx}\left(\frac{4x^2+3}{3x^2-2}\right)=\frac{(3x^2-2).8x-(4x^2+3).6x}{(3x^2-2)^2}=\frac{-34x}{(3x^2-2)^2}$$

10. By Newton's Raphson, the positive root of $x^3-18=0$ in (2, 3) is

(a) 2.666 (b) 2.621 (c) 2.620 (d) 2.622

✍ (a) 2.621

Explanation: Use calculator as in https://youtu.be/xFXG8G0IV9I

11. Two forces acting at an angle of 45° have a resultant equal to $\sqrt{10}N$, if one of the forces be $\sqrt{2}N$, what is the other force.

(a) 1 N (b) 2 N (c) 3 N (d) 4 N

OR

The total cost function of a producer is given as $C=500+30Q+\frac{1}{2}Q^2$. What is the marginal cost (MC) at $Q=4$ is

(a) Rs. 38 (b) Rs. 34 (c) Rs. 30 (d) Rs. 28

✍ (b) 2N

Explanation:

$$R^2=P^2+Q^2+2PQ\cos\alpha$$

$\Rightarrow 8=Q^2+2Q$

$\Rightarrow Q=2$ (Plug in the given option)

✍ OR (b) Rs. 34

Explanation:

$$MC=\frac{dC}{dQ}=\frac{d}{dQ}\left(500+30Q+\frac{1}{2}Q^2\right)=30+Q$$

$\Rightarrow MC=34$ at $Q=4$

Group 'B' **(5 × 8 = 40)**

12. A function $f(x) = x^2$ is given. Answer the following question for the function $f(x)$.

(i) What is the algebraic nature of the function?

(ii) Write the name of the locus of the curve.

(iii) Write the vertex of the function.

(iv) Write any one property for sketching the curve.

(v) Write the domain of the function.

Let $y = f(x) = x^2$

(i) The algebraic nature of the function $f(x) = x^2$ is quadratic.

(ii) The name of the locus of the curve $y = x^2$ is parabola.

(iii) The vertex of the function $y = x^2$ is (0, 0).

(iv) Here, $f(-x) = (-x)^2 = x^2 = f(x)$. Hence, $f(x) = x^2$ is an even function.

(v) The given function is defined for all $x \in R$, so domain of $f = R = (-\infty, \infty)$.

13. Compare the sum of n terms of the series: $1 + 2a + 3a^2 + 4a^3 + ...$ and $a + 2a + 3a + 4a + ...$ up to n terms

Series 1: $1 + 2a + 3a^2 + 4a^3 + ...$

Here, series 1 is arithmetico-geometric series.

Let $S_{n1} = 1 + 2a + 3a^2 + 4a^3 + ... + (n-1)a^{n-2} + na^{n-1}$

Then, $aS_{n1} = \quad a + 2a^2 + 3a^3 + ... + (n-1)a^{n-1} + na^n$

$-\quad\quad - \quad - \quad - \quad\quad - \quad\quad -$

By subtraction, we get

$$(1-a)S_{n1} = 1 + a + a^2 + a^3 + ... + a^{n-1} - na^n$$

or, $$(1-a)S_{n1} = \frac{1-a^n}{1-a} - na^n$$

$\therefore$ $$S_{n2} = \frac{1-a^n}{(1-a)^2} - \frac{na^n}{1-a}$$

Series 2: $a + 2a + 3a + 4a + ...$

Here, series 2 is arithmetic series.

Let $S_{n2} = a + 2a + 3a + 4a + ... + na$

$$= \frac{n}{2}[2.a + (n-1).a]$$

$$= \frac{n(2a + na - a)}{2}$$

$$= \frac{n(na + a)}{2}$$

$$= \frac{n(n+1)a}{2}$$

14. a) In any triangle, prove that: $(b+c)\sin\frac{A}{2} = a\sin\left(\frac{A}{2} + B\right)$ **(3)**

We know,

$$\begin{aligned} b + c &= 2R\sin B + 2R\sin C \\ &= 2R(\sin B + \sin C) \\ &= 2R.2\sin\frac{B+C}{2}.\cos\frac{B-C}{2} \\ &= 4R\sin\frac{B+C}{2}\cos\frac{B-C}{2} \qquad ......(1) \end{aligned}$$

And,

$$\begin{aligned} a &= 2R\sin A \\ &= 2R\sin[180^o - (B+C)] \\ &= 2R\sin(B+C) \\ &= 2R.2\sin\frac{B+C}{2}\cos\frac{B+C}{2} \\ &= 4R\sin\frac{B+C}{2}\cos\frac{B+C}{2} \qquad ......(2) \end{aligned}$$

Dividing (1) by (2), we get

$$\frac{b+c}{a} = \frac{4R\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{4R\sin\frac{B+C}{2}\cos\frac{B+C}{2}}$$

$$= \frac{\cos\frac{B-C}{2}}{\cos\frac{B+C}{2}}$$

$$= \frac{\cos\frac{B-[180^o-(A+B)]}{2}}{\cos\frac{180^o - A}{2}}$$

$$= \frac{\cos\frac{B - 180^o + A + B}{2}}{\cos\left(90^o - \frac{A}{2}\right)}$$

or, $\dfrac{b+c}{a}=\dfrac{\cos\dfrac{A+2B-180^o}{2}}{\cos\left(90^o-\dfrac{A}{2}\right)}$

or, $\dfrac{b+c}{a}=\dfrac{\cos\left(\dfrac{A}{2}+B-90^o\right)}{\cos\left(90^o-\dfrac{A}{2}\right)}$

or, $\dfrac{b+c}{a}=\dfrac{\sin\left(\dfrac{A}{2}+B\right)}{\sin\dfrac{A}{2}}$

$\therefore$ $(b+c)\sin\dfrac{A}{2}=a\sin\left(\dfrac{A}{2}+B\right)$. Hence, proved.

14. b) Express $\vec{r}=(4,7)$ as the linear combination of $\vec{a}=(5,-4)$ and $\vec{b}=(-2,5)$ (2)

Let $\vec{r}=x\vec{a}+y\vec{b}$ where x and y are scalars.

Then, $(4,7)=x(5,-4)+y(-2,5)$

or, $(4,7)=(5x,-4x)+(-2y,5y)$

or, $(4,7)=(5x-2y,-4x+5y)$

Equating corresponding elements, we get

$5x-2y=4$ ...(1)

$-4x+5y=7$ ...(2)

Multiplying (1) by 5 and (2) by 2 and adding them, we get

$$\begin{array}{rl} 25x-\cancel{10y} & =20 \\ -8x+\cancel{10y} & =14 \\ + \quad & \quad + \\ \hline 17x \quad & =34 \end{array}$$

or, $x=\dfrac{34}{17}$

$\therefore$ $x=2$

Substituting the value of x in (2), we get

$-4.2+5y=7$

or, $5y=15$

or, $y=\dfrac{15}{5}$

$\therefore$ $y=3$

Hence, the required linear combination is $\vec{r}=2\vec{a}+3\vec{b}$

15. Calculate the appropriate measure of skewness for the data below.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No of workers	10	12	25	35	40	50

✍ Since the given distribution is not open ended, it is appropriate to use Karl Pearson's coefficient as below:

Class	Mid-value (x)	No. of workers (f)	$d' = \frac{x-25}{10}$	fd'	fd'^2	c.f.
0 – 10	5	10	– 2	– 20	40	10
10 – 20	15	12	– 1	– 12	12	22
20 – 30	25	25	0	0	0	47
30 – 40	35	35	1	35	35	82
40 – 50	45	40	2	80	160	122
50 – 60	55	50	3	150	450	172
		$N = 172$		$\sum fd' = 233$	$\sum fd'^2 = 697$	

Here, $a = 25$, $N = 172$, $\sum fd' = 233$, $\sum fd'^2 = 697$, $h = 10$

We know,

$$\bar{x} = a + \frac{\sum fd'}{N} \times h = 25 + \frac{233}{172} \times 10 = 38.55$$

And,

$$\sigma = h\sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} = 10\sqrt{\frac{697}{172} - \left(\frac{233}{172}\right)^2} = 14.89$$

The given distribution is unsymmetrical, so

$$S_k(P) = \frac{3(\bar{x} - M_d)}{\sigma}$$

So, first we find M_d

$$\frac{N}{2} = \frac{172}{2} = 86 \text{ which lies in the class } 40 - 50$$

Here, $l = 40, \frac{N}{2} = 86, c.f. = 82, f = 40, h = 10$

$$M_d = l + \frac{\frac{N}{2} - c.f.}{f} \times h = 40 + \frac{86 - 82}{40} \times 10 = 41$$

Now,

$$S_k(P) = \frac{3(\bar{x} - M_d)}{\sigma} = \frac{3(38.55 - 41)}{14.89} = -0.49$$

Hence, the distribution is negatively skewed.

16. Define different types of discontinuity of a function. Also write the condition for increasing, decreasing and concavity of function. (2 + 3)

First Part:

A discontinuous function may be of the following types:

i) If $\lim_{x\to a} f(x)$ does not exist *i.e.*, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ then the function $f(x)$ is said to be an ***ordinary discontinuity***.

ii) If $\lim_{x\to a} f(x) \neq f(x)$ then the function $f(x)$ is said to have a ***removable discontinuity*** at $x = a$.

iii) If $\lim_{x\to a} f(x) \to \infty$ or $-\infty$ then the function $f(x)$ is said to have ***infinite discontinuity*** at $x = a$.

Second Part:

Function $y = f(x)$	Condition
Increasing function	$\frac{dy}{dx} = f'(x) > 0$
Decreasing function	$\frac{dy}{dx} = f'(x) < 0$
Concavity of function *i)* Concave upward	$\frac{d^2y}{dx^2} = f''(x) > 0$
ii) Concave downward	$\frac{d^2y}{dx^2} = f''(x) < 0$

17. Evaluate: $\int \frac{x^2dx}{\sqrt{a^2 - x^2}}$

Let $x = a\sin\theta$ then $dx = a\cos\theta d\theta$ and $a^2 - x^2 = a^2 - a^2\sin^2\theta = a^2\cos^2\theta$

$$\therefore \int \frac{x^2dx}{\sqrt{a^2-x^2}} = \int \frac{a^2\sin^2\theta . a\cos\theta d\theta}{\sqrt{a^2\cos^2\theta}}$$

$$= \int \frac{a^2\sin^2\theta . a\cos\theta d\theta}{a\cos\theta}$$

$$= a^2\int \sin^2\theta d\theta$$

$$= a^2\int \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{a^2}{2}\int d\theta - \frac{a^2}{2}\int \cos 2\theta d\theta$$

$$= \frac{a^2}{2}\theta - \frac{a^2}{2}.\frac{\sin 2\theta}{2} + c$$

$$= \frac{a^2}{2}\theta - \frac{a^2}{2}.\frac{2\sin\theta\cos\theta}{2} + c$$

$$= \frac{a^2}{2}\theta - \frac{a^2}{2}\sin\theta\sqrt{1-\sin^2\theta} + c$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{a^2}{2}.\frac{x}{a}.\sqrt{1-\frac{x^2}{a^2}} + c$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + c$$

18. Define Trapezoidal rule. Evaluate using Trapezoidal rule for $\int_0^1 \frac{dx}{1+x}$, $n = 4$

✍ **Trapezoidal Rule:**

If a function f is continuous on the closed interval $[a, b]$, then

$$\int_a^b f(x)dx \approx \frac{(b-a)}{2n}[f(x_o)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]$$

where the closed interval $[a, b]$ has been partitioned into n sub-intervals $[x_o, x_1]$, $[x_1, x_2]$,..., $[x_{n-1}, x_n]$, each of length $\frac{(b-a)}{n}$.

Next Part:

Since $n = 4$, $h = \frac{(b-a)}{n} = \frac{1-0}{4} = 0.25$ and the five points to be considered are $x_o = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $x_4 = 1$. Evaluating the values of the function at these points, we get

End point	$x_o = 0$	$x_1 = 0.25$	$x_2 = 0.5$	$x_3 = 0.75$	$x_4 = 1$
$y = \frac{1}{1+x}$	1.00000	0.80000	0.66666	0.57143	0.50000

Now, using trapezoidal rule

$$\int_0^1 \frac{1}{1+x}dx = \frac{h}{2}[y_o + 2y_1 + 2y_2 + 2y_3 + y_4]$$
$$= \frac{0.25}{2}[1.00000 + 2\times 0.80000 + 2\times 0.66666 + 2\times 0.57143 + 0.50000]$$
$$= 0.1250 \times 5.57618$$
$$= 0.69702$$

19. State sine law and use it to prove Lami's theorem.

OR

A decline in the price of good *X* by Rs. 5 causes an increase in its demand by 20 units to 50 units. The new price of *X* is Rs. 15.

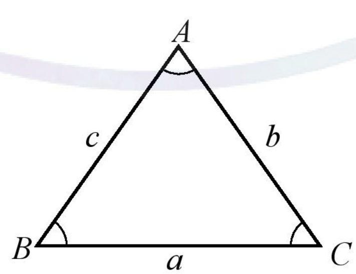
(i) **Calculate elasticity of demand.**

(ii) **The elasticity of demand is negative, what does it mean?**

✍ **Sine Law:**

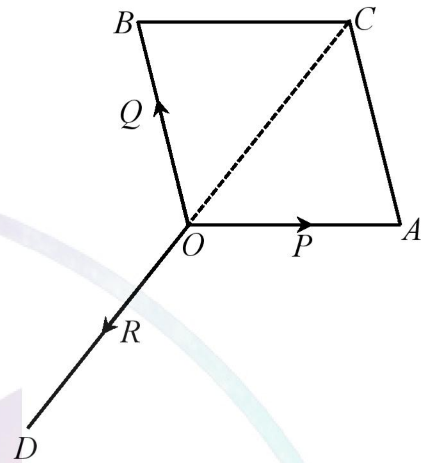
In any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Lami's Theorem:

If three forces acting at a point, be equilibrium, each force is proportional to the sine of the angle between the other two.



Let P, Q and R be three forces acting at equilibrium point O, represented OA, OB and OD respectively. Complete the parallelogram $OACB$ in which diagonal OC represents the resultant of forces P and Q. Since the forces are in equilibrium, the resultant of forces P and Q represented by OC will be balanced by the force R. That is, the force represented by OC is equal and opposite to the force R. So, CO represents the force R. Since AC and OB are equal and parallel, so AC represents the force Q.

In triangle OAC, using Sine Law

$$\frac{OA}{\sin OCA} = \frac{AC}{\sin COA} = \frac{CO}{\sin OAC}$$

$$\text{or, } \frac{P}{\sin OCA} = \frac{Q}{\sin COA} = \frac{R}{\sin OAC}$$

$$\text{Also, } \sin OCA = \sin COB = \sin(180^o - QOR) = \sin QOR$$

$$\sin COA = \sin(180^o - ROP) = \sin ROP$$

$$\sin OAC = \sin(180^o - POQ) = \sin POQ$$

$$\therefore \frac{P}{\sin QOR} = \frac{Q}{\sin ROP} = \frac{R}{\sin POQ}$$

OR Part

Here, $\Delta P = -\text{Rs. } 5,\ Q_1 = 20 \text{ units},\ Q_2 = 50 \text{ units},\ P_2 = \text{Rs. } 15$

Then, $\Delta Q = Q_2 - Q_1 = 50 - 20 = 30 \text{ units}$

And, $P_1 = P_2 - \Delta P = \text{Rs. } 15 - (-\text{Rs. } 5) = \text{Rs. } 20$

i) Elasticity of demand $= \frac{\Delta Q}{\Delta P} \times \frac{P_1}{Q_1}$

$$= \frac{30}{-5} \times \frac{20}{20}$$

$$= -6$$

ii) The elasticity of demand is negative means there is inverse relationship between price and quantity demanded i.e., demand will increase when the price decreases and demand will decrease when price increases.

Group 'C' **(8× 2 = 24)**

20. (a) The factor of expression $\omega^3 - 1$ are $\omega - 1$ and $\omega^2 + \omega + 1$. If $\omega^3 - 1 = 0$

(i) **Find the possible values of ω and write the real and imaginary roots of ω.** **(2)**

(ii) **Prove that: $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix} = 0$ where n is positive integer.** **(4)**

✍ **(*i*) Part:**

Here, $\omega^3 - 1 = 0$

or, $(\omega - 1)(\omega^2 + \omega + 1) = 0$

Either $\omega - 1 = 0 \Rightarrow \omega = 1$

or, $\omega^2 + \omega + 1 = 0 \Rightarrow \omega = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow \omega = \dfrac{-1 \pm \sqrt{-3}}{2}$

$\Rightarrow \omega = \dfrac{-1 \pm \sqrt{3}i}{2}$

Hence, the possible value of ω are 1, $\dfrac{-1+\sqrt{3}i}{2}$ and $\dfrac{-1-\sqrt{3}i}{2}$.

(*ii*) Part:

$$LHS = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \end{vmatrix} \qquad C_1 \to C_1 + C_2 + C_3$$

$$= (1+\omega^n+\omega^{2n})\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ 1 & 1 & \omega^n \\ 1 & \omega^{2n} & 1 \end{vmatrix} \qquad \text{Taking } 1+\omega^n+\omega^{2n} \text{ common from } C_1$$

$$= (1+\omega^n+\omega^{2n})\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ 0 & 1-\omega^n & \omega^n-\omega^{2n} \\ 0 & \omega^{2n}-\omega^n & 1-\omega^{2n} \end{vmatrix} \qquad R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1$$

$$= (1+\omega^n+\omega^{2n})\left[(1-\omega^n)(1-\omega^{2n}) - (\omega^{2n}-\omega^n)(\omega^n-\omega^{2n})\right]$$

$$= (1+\omega^n+\omega^{2n})\left[1-\omega^{2n}-\omega^n+\omega^{3n}+(\omega^n-\omega^{2n})^2\right]$$

$$= (1+\omega^n+\omega^{2n})\left[1-\omega^{2n}-\omega^n+\omega^{3n}+\omega^{2n}-2\omega^{3n}+\omega^{4n}\right]$$

$$= (1+\omega^n+\omega^{2n})(1-\omega^n-\omega^{3n}+\omega^{4n})$$

$= (1+\omega^n+\omega^{2n})(1-\omega^n-1+\omega^n)$ $\quad \because \omega^3 = 1 \Rightarrow \omega^{3n} = 1$ and $\omega^{4n} = \omega^n$

$= (1+\omega^n+\omega^{2n}).0$

$= 0$

$= RHS$

20. (b) Verify that: $|x+y| \le |x| + |y|$ with $x = 2$ and $y = -3$ **(2)**

Here, $x = 2$ and $y = -3$

$|x+y| = |2-3| = |-1| = 1$

$|x|+|y| = |2|+|-3| = 2+3 = 5$

$\therefore |x+y| \le |x|+|y|$

21. (a) The single equation of pair of lines is $2x^2+3xy+y^2+5x+2y-3=0$

(i) **Find the equation of pair of straight lines represented by the single equation.** **(4)**

(ii) **Are the pair of lines represented by the given equation passes through origin? Write with reason.** **(1)**

(iii) **Find the point of intersection of the pair of lines.** **(2)**

***(i)* Part:**

Here, the given equation is $2x^2+3xy+y^2+5x+2y-3=0$ ...(1)

The given equation can be written as

$$y^2+(3x+2)y+(2x^2+5x-3)=0$$

This is quadratic in y. Solving for y, we get

$$y = \frac{-(3x+2) \pm \sqrt{(3x+2)^2-4(2x^2+5x-3)}}{2}$$

$$= \frac{-(3x+2) \pm \sqrt{9x^2+12x+4-8x^2-20x+12}}{2}$$

$$= \frac{-(3x+2) \pm \sqrt{x^2-8x+16}}{2}$$

$$= \frac{-(3x+2) \pm \sqrt{(x-4)^2}}{2}$$

$$= \frac{-(3x+2) \pm (x-4)}{2}$$

Taking – ve sign,

$$y = \frac{-(3x+2)-(x-4)}{2}$$

or, $y = \dfrac{-4x+2}{2}$

or, $y = -2x+1$

$\therefore \quad 2x+y-1=0$

Taking +ve sign,

$$y = \frac{-(3x+2)+(x-4)}{2}$$

or, $y = \dfrac{-2x-6}{2}$

or, $y = -x-3$

$\therefore \quad x+y+3=0$

Thus, the separate equations are:

$2x+y-1=0 \qquad \ldots(2)$

$x+y+3=0 \qquad \ldots(3)$

***ii*) Part:**

The pair of line represented by the given equation does not pass-through origin because the given equation is not homogenous.

***iii*) Part:**

Subtracting (3) from (2), we get

$$\begin{array}{r} 2x+\cancel{y}-1=0 \\ x+\cancel{y}+3=0 \\ - \quad - \quad - \\ \hline x \qquad -4=0 \end{array}$$

$\therefore x=4$

Substituting the value of x in (3), we get

$4+y+3=0$

$\therefore y=-7$

Hence, the point of intersection of the pair of lines is $(4,-7)$

21 (b) If three vectors $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors in space then write a relation between them. (1)

✍ If three vectors $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors in space then

$\vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

$\vec{b}.\vec{c} = \vec{c}.\vec{b} = 0$

$\vec{c}.\vec{a} = \vec{a}.\vec{c} = 0$

$\vec{a}.\vec{a} = \vec{b}.\vec{b} = \vec{c}.\vec{c} = 1$

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{3}$

22. (*i*) Distinguish between derivative and anti-derivative of a function. Write their physical meanings and illustrate with example in your context. Find the differential coefficient of log(sin*x*) with respect to *x*. **(1 + 2+ 2)**

Derivative	Anti-derivative
The derivative of a function $y = f(x)$ at a point x is defined as $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$, provided the limit exits.	An antiderivative is the opposite of a derivative. An antiderivative of a function $f(x)$ is a function whose derivative is equal to $f(x)$. That is, if $F'(x) = f(x)$ then $F(x)$ is an antiderivative of $f(x)$.
The derivative of a function $y = f(x)$ with respect to x is denoted as $f'(x), y', \frac{df}{dx}, \frac{dy}{dx}$	The anti-derivative of a function $f(x)$ is denoted by $\int f(x)dx$

Physically, the derivative is defined as an instantaneous rate of change at a given point. For example, the instantaneous velocity $v(t)$ is the derivative of the position function $s(t)$.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

That is, $v(t) = s'(t)$. Furthermore, the acceleration $a(t)$ is the derivative of the velocity $v(t)$-that is, $a(t) = v'(t) = s''(t)$. Now suppose we are given an acceleration function $a(t)$, but not the velocity function v(t) or the position function s(t). Since $a(t) = v'(t)$ determining the velocity function requires us to find an antiderivative of the acceleration function. Then, since $v(t) = s'(t)$ determining the position function requires us to find an antiderivative of the velocity function.

Let $y = \log(\sin x)$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d\log(\sin x)}{dx}$$

$$= \frac{d\log(\sin x)}{d(\sin x)} . \frac{d(\sin x)}{dx}$$

$$= \frac{1}{\sin x} . \cos x$$

$$= \cot x$$

Hence, the differential coefficient of log (sin x) is cot x.

22. (*ii*) Find the area bounded by the *y*-axis, the curve $x^2 = 4(y-2)$ and the line $y = 11$. (3)

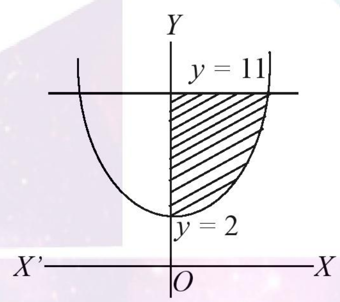
Here, $x^2 = 4(y-2)$

$\therefore x = \sqrt{4(y-2)} = 2\sqrt{y-2}$

The curve $x^2 = 4(y-2)$ meets the y axis at the point where $x = 0$.

so, $4(y-2) = 0$
or, $y - 2 = 0$
$\therefore \quad y = 2$

Now,



$$A = \int_2^{11} x dy$$
$$= \int_2^{11} 2\sqrt{y-2}\, dy$$
$$= 2\int_2^{11} (y-2)^{\frac{1}{2}} dy$$
$$= 2\left[\frac{1}{\frac{1}{2}+1}(y-2)^{\frac{1}{2}+1}\right]_2^{11}$$
$$= 2\left[\frac{1}{\frac{3}{2}}(y-2)^{\frac{1}{2}+1}\right]_2^{11}$$
$$= 2\left[\frac{2}{3}(y-2)^{\frac{3}{2}}\right]_2^{11}$$
$$= \frac{4}{3}\left[(11-2)^{\frac{3}{2}} - (2-2)^{\frac{3}{2}}\right]$$
$$= \frac{4}{3}.(9)^{\frac{3}{2}}$$
$$= \frac{4}{3}.27$$
$$= 36$$

Hence, the required area is 36 sq. units.